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# The influences of velocity on pressure losses in hydrate slurry



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## Abstract

**Purpose** – In recent years, hydraulic transfer in solid-liquid flows has been paid considerable attention by many researchers. One of the most important parameters of this kind of flow is the pressure losses. This paper aims to focus on this aspect.

**Design/methodology/approach** – Two- and three-layer models introduced by Doron are significantly capable of predicting the influences of slurry velocity on the pressure losses. In this research paper, using numerical methods, the related equations are solved for two- and three-layer models and the influence of the slurry velocity on the pressure losses is studied.

**Findings** – The results show that, as long as there is a stable bed at the bottom of the pipe, the pressure losses are independent of the slurry velocity.

**Originality/value** – When the height of the stable bed becomes zero (when the stable bed disappears), the pressure losses increase considerably as the slurry velocity increases.


**Keywords** Fluid pressure, Flow

**Paper type** Research paper

## Nomenclature

| | |
|---|---|
| $A_h$ | = Cross section area of the upper layer ($m^2$) |
| $A_{mb}$ | = Cross section area of the mobile bed ($m^2$) |
| $A_{sb}$ | = Cross section area of the stable bed ($m^2$) |
| $C_s$ | = Concentration of the fluid |
| $C_h$ | = Concentration of the upper layer |
| $C_{mb}$ | = Concentration of the mobile bed |
| $C_D$ | = Drag coefficient |
| $D$ | = Pipe diameter (m) |
| $dp/dx$ | = Pressure gradient (pa/m) |
| $d_p$ | = Diameter of solid particles (m) |
| $\varepsilon$ | = Slurry particles diffusion coefficient ($m^2/s$) |
| $F_{sb}$ | = Stable bed friction force ($N$) |
| $F_{mb}$ | = Friction force exerted onto $A_{mb}$ ($N$) |
| $F_{mbsb}$ | = Friction force exerted onto $A_{sb}$ ($N$) |
| $F_{mbsb}$ | = Friction force exerted from $A_{mb}$ onto $A_{sb}$ (or reverse) ($N$) |
| $g$ | = Gravitational acceleration ($m/s^2$) |
| $\theta_{mb}$ | = Central angle of the mobile bed |
| $\theta_{sb}$ | = Central angle of the stable bed |
| $\rho_s$ | = Particle density ($kg/m^3$) |
| $\rho_l$ | = Liquid density ($kg/m^3$) |
| $S_h$ | = Surface area of the upper layer ($m_2$) |
| $S_{hmb}$ | = Surface area of the interface of the mobile bed and upper layer ($m^2$) |
| $S_i$ | = Surface area of the interface of the two layers ($m^2$) |
| $S_{mb}$ | = Surface area of the mobile bed ($m^2$) |
| $S_{mbsb}$ | = Surface area of the interface of mobile bed and the stable bed ($m^2$) |
| $\tau_h$ | = Hydraulic shear stress affecting the environment of the mobile bed (Pa) |
| $\tau_i$ | = Shear stress exerted on the interface of the two layers (Pa) |
| $\tau_{mb}$ | = Shear stress exerted on mobile bed perimeter (Pa) |
| $\tau_{hmb}$ | = Shear stress exerted at the interface of the mobile and upper layer (Pa) |
| $\tau_{mbsb}$ | = Shear stress exerted at the interface of the mobile and stable bed (Pa) |
| $U_h$ | = Mean velocity of the upper layer (m/s) |
| $U_{mb}$ | = Mean velocity of the mobile bed (m/s) |
| $U_S$ | = Mean velocity of fluid (m/s) |

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|---|---|---|---|
| $V_h$ | = Volume covered by the upper layer ($m^3$) | $y_{mb}$ | = Thickness of the mobile bed (m) |
| $w$ | = Solid particle settling velocity (m/s) | $y_{sb}$ | = Thickness of the stable bed (m) |
| | | $g$ | = Gravitational acceleration ($m/s^2$) |

## 1. Introduction

The hydrate slurry flows, containing floating solid particles have complicated properties and are totally different from the ones of the one-phase flows. Different series of researches have been done on these flows and also a wide range of them were done on the prediction of pressure losses in slurry flows by scientists such as Gillies and Shook (1991), Doron and Barnea(1995), Wasp *et al.* (1979) and Wilson and Pulgh (1988). Knowledge of the effective parameters on slurry flow such as pressure losses and terminal deposit velocity is of great important in systems containing solid-liquid flows. Using slurry flows, in the water sewer, water sewerage, water purifications and design systems. The two layers model introduced by Wasp and the one resulted by Doron's researches are not able to correctly estimate the pressure losses in commercial slurry flow pipes. Efforts were made to develop these models to better predict the pressure losses. Of the introduced models which optimized Doron two layers model was Doron and Barnea (1993) three layers model in 1993. The model is notably able to illustrate the influences of different parameters on the pressure losses in slurry flows.

Getting to know the effective parameters on slurry flows such as pressure losses seem to be of great importance in designing systems containing solid-liquid flows. Since, the basic fluid equations are not capable of predicting the properties of this type of flows, attempts were made to model these properties using the introduced models, and to predict them through these models.

Predicting the pressure losses for hydrate slurry in horizontal pipes is very important in designing systems running with this sort of slurry. This sort of slurry consists tiny particles and its properties are different from the ones studied in recent years. Solving the three-layer model and comparing the results with the previous experimental ones, this model can be used to predict the pressure losses in hydrate slurry.

## 2. The problem definition

Let us consider a solid-liquid flow in a horizontal pipe. If there is a high-mass flux, the particles will be suspended. If the flux decreases, since the particles density is greater, they will tend to make deposit at the bottom of the pipe and form a stable bed. Based on this theory, Doron and Barnea (1993) introduced a two-layer model to study this kind of flow in horizontal pipes in 1993. They have divided slurry flows, in horizontal pipes, into two different categories as follows:

(1) The heterogeneous flow in the upper part of the pipe with a steady contribution of particles.

(2) The lower part flow with a non-homogenous contribution of particles which carries out most of the particle transfer.

According to the model, when the exerted force to move the particles in the moving bed (the inertial force) is less than the opposing force exerted onto the particles (the particles weight), some of the particles stop moving. This model is very useful in case there is no stable bed and the results are compatible with experimental ones. A schematic view of the model is shown in Figure 1.

*2.1 Continuity equations*
Assuming that the flow in each layer is defined by the average properties of the contained materials, the continuity equations for the heterogeneous layer and stable bed are written as follows.

The continuity equation for the solid phase:

$$U_h C_h A_h + U_{\mathrm{mb}} C_{\mathrm{mb}} A_{\mathrm{mb}} = U_S C_S A_{\mathrm{sb}} \tag{1}$$

The continuity equation for the liquid phase:

$$U_h(1 - C_h)A_h + U_{\mathrm{mb}}(1 - C_{\mathrm{mb}})A_{\mathrm{mb}} = U_S(1 - C_S)A_{\mathrm{sb}} \tag{2}$$

*2.2 Momentum equations*
The flow properties of the upper heterogeneous layer remain unchanged as it contains solid particles. Therefore, the balancing of the forces for this layer is similar to the fluid flow as follows (Doron *et al.*, 1986):

$$V_h \frac{\mathrm{d}p}{\mathrm{d}x} = -\tau_h s_h - \tau_i s_i \tag{3}$$

The force balance for the stable bed containing a non-homogenous contribution of particles is:

$$V_h \frac{\mathrm{d}p}{\mathrm{d}x} = -F_{\mathrm{sb}} - \tau_{\mathrm{mb}} s_{\mathrm{mb}} + \tau_i s_i \tag{4}$$

$\mathrm{d}p/\mathrm{d}x$ is a phrase representing the pressure losses caused by slurry motion in Pascal per length unit of pipe Pa/m.

*2.3 Diffusion equations*
Doron assumed that the distribution of particles within the upper heterogeneous layers is resulted by the diffusion equation which is studied in the subsequent sections (Doron and Barnea, 1995):

$$\varepsilon \frac{d^2 c}{dy^2} + w \frac{\mathrm{d}c}{\mathrm{d}y} = 0 \tag{5}$$

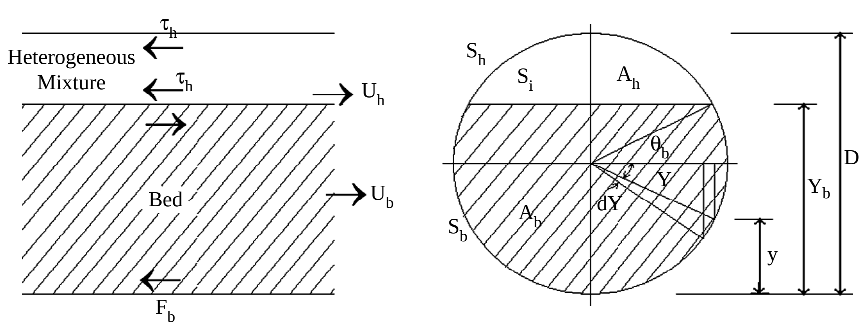



**Figure 1.**
A schematic view of the two-layer model

In this equation, $y$ is the perpendicular direction to the pipe axis and $w$ is the terminal deposit velocity which can be attained by balancing the gravity and drag forces exerted to the particles, $c$ is the volumetric concentration of solid particles and $\varepsilon$ is the diffusion coefficient that can be obtained from the equation introduced by Taylor (1954). The diffusion coefficient of turbulent flow and the flow with Reynolds number below than 2,000 is very similar:

$$\varepsilon = 0.052 \ U_* r \tag{6}$$

$U_*$ is the shear velocity and $r$ is the hydraulic radius of the upper heterogeneous layer cross section area. The shear velocity is obtained by the equation:

$$U_* = U_h \sqrt{\frac{f_i}{2}} \tag{7}$$

$f_i$ is the friction factor, and the terminal deposit velocity is obtained by Stocks as the following equation (Richardson and Zaki, 1954):

$$w_o = \sqrt{\frac{2(\rho_s - \rho_l) g V_S}{C_D A_{\mathrm{sb}} \rho}} \tag{8}$$

In order to obtain the final terminal deposit velocity of the larger particles, $w$, equation (8) was modified by Richardson and Zaki (1954) as follows:

$$\frac{w}{w_o} = (1 - C_h)^m \tag{9}$$

$m$ is 2.36 for $Re > 500$, where $Re$ is the Reynolds number of the particles related to their terminal velocity $w$.

Doron overlooked the concentration changes in the horizontal direction and considered it one dimensional. Solving the equation on the cross section area of the pipe, the particles concentration distribution equation used by Doron is obtained as follows (Doron and Barnea, 1995):

$$\frac{C_h}{C_{\mathrm{mb}}} = \frac{2(D/2)^2}{A_h} \int \exp\left(-\frac{wD}{2\varepsilon}\left[\sin\gamma - \sin\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{2y}{D} - 1\right)\right)\right]\right) \cos^2\gamma \cdot d\gamma \tag{10}$$

Therefore, considering equations (1)-(4) and (10) and solving these nonlinear equations, the following unknowns are obtained:

- the average velocity of the upper layers, $U_h$;
- the average velocity of the moving bed, $U_{\mathrm{mb}}$;
- the average concentration of the upper layer, $C_h$;
- the height of the moving bed, $y$; and
- the pressure gradient (for the length unit of the pipe), $\mathrm{d}p/\mathrm{d}x$.

## 3. Doron three-layer model

This model divides the flow into three parts in case of low velocity: the stable bed at the bottom of the pipe, the moving bed on the former and the last one which is a heterogeneous suspension of particles. If the velocity of the moving bed reaches a minimum, the solid particles tend to deposit and if the velocity goes less than that, a stable bed with a certain height is produced. Therefore, balancing the forces exerted to the particles of the layers beneath the moving bed and the layers above the stable bed (exactly on the border of the stable and moving beds), the minimum velocity to produce this layer can be resulted as follows (Doron *et al.*, 1986; Figure 2):

$$U_{\mathrm{bc}} = \sqrt{\frac{0.779(\rho_s - \rho_l) g d_p \left[ C_{\mathrm{mb}} \frac{y_{\mathrm{mb}}}{d_p} + (1 - C_{\mathrm{mb}}) \right]}{\rho_l C_D}} \tag{11}$$

The balancing of the forces (momentum equations) is written parallel to slurry motion direction. The heterogeneous mixture of the upper layers functions as a liquid net:

$$V_h \frac{\mathrm{d}p}{\mathrm{d}x} = -\tau_h S_h - \tau_{\mathrm{hmb}} S_{\mathrm{hmb}} \tag{12}$$

where $\mathrm{d}p/\mathrm{d}x$ is the pressure gradient. For the moving bed, the force balance leads to:

$$V_{\mathrm{mb}} \frac{\mathrm{d}p}{\mathrm{d}x} = -F_{\mathrm{mbsb}} - \tau_{\mathrm{mbsb}} S_{\mathrm{mbsb}} - F_{\mathrm{mb}} - \tau_{\mathrm{mb}} S_{\mathrm{mb}} + \tau_{\mathrm{hmb}} S_{\mathrm{hmb}} \tag{13}$$

The balancing of the forces on the stable bed is not part of the problem and the inequality which studies the stable bed is as follows:

$$V_{\mathrm{sb}} \frac{\mathrm{d}p}{\mathrm{d}x} = +F_{\mathrm{mbsb}} + \tau_{\mathrm{mbsb}} S_{\mathrm{mbsb}} \leq F_{\mathrm{sb}} \tag{14}$$

The mechanism of the distribution of the particles in the upper beds is stated by diffusion equation and integrating this equation on the cross section area of the upper beds, brings about the average concentration as:

$$\frac{C_h}{C_{\mathrm{mb}}} = \frac{2(D/2)^2}{A_h} \int_{\theta_{\mathrm{mb}}+\theta_{\mathrm{sb}}}^{\pi/2} \exp\left( -\frac{wD}{2\varepsilon} [\sin\gamma - \sin(\theta_{\mathrm{mb}} + \theta_{\mathrm{sb}})] \right) \cos^2\gamma \, d\gamma \tag{15}$$

All the terms included in the above equation are written and solved by average velocity parameters of the upper layers $U_h$, the average velocity of the moving bed $U_{\mathrm{mb}}$, the

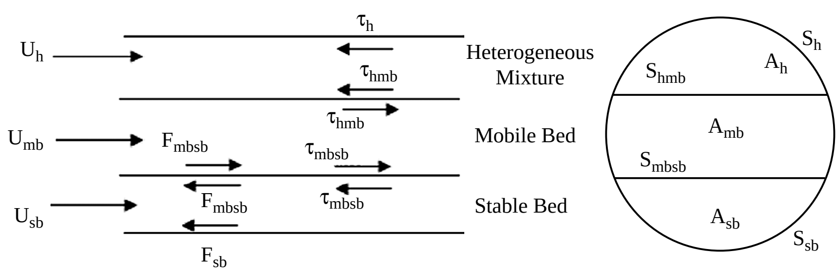



**Figure 2.**
The analysis of shear stress in the three-layer model

average concentration of the upper beds $C_h$, the moving bed height $y_{mb}$, the stable bed height $y_{sb}$ and the pressure gradient $dp/dx$.

The governing equation in this work are solved numerically MathCAD software and the numerical solution studied in this paper and Doron solution and the experimental results are to be compared with each other. Then the influence of the slurry velocity on the pressure losses is to be studied and then this model is used to determine design parameters in hydrate slurry.

Figure 3 shows the present prediction of the terminal deposit velocity and experimental and theoretical data obtained by Doron and Barnea (1995). It is clearly seen that the terminal deposit velocity decreases relative to the increase in the concentration of the dilute solutions and after a maximum value, decreases with light slope.

## 4. The pressure losses

One of the important properties of a flow is the relation between the pressure losses and the flux. In Figure 4, the influences of volumetric concentration on the pressure gradient of the two- and three-layer models for different concentrations are compared.

The terminal deposit velocity conditions can be determined by the breaks on "pressure loss – flux" diagram (where the curve slope goes through sudden changes). This shows that the three-layer model is obtained, base upon an enormous experimental data bank. Therefore, in low fluxes, a stable bed is predicted by the three-layer model and the pressure losses are independent of the flux. These observations are compatible with the experimental ones. Since, low-flux flows are not usually steady, little data can be obtained considering the stable bed, but this data are enough to predict the pressure losses in low-flux flows.

In Figures 4 and 5, in a range of velocity where there is a stable bed at the bottom of the pipe, pressure losses are independent of the flow velocity, whereas at velocities higher than the terminal deposit velocity, the higher the flow velocity, the higher is the pressure loss rate.

Figure 6 can be seen that as the volumetric concentration increases, the terminal deposit velocity decreases. This fact is due to the mutual influence of the particles in

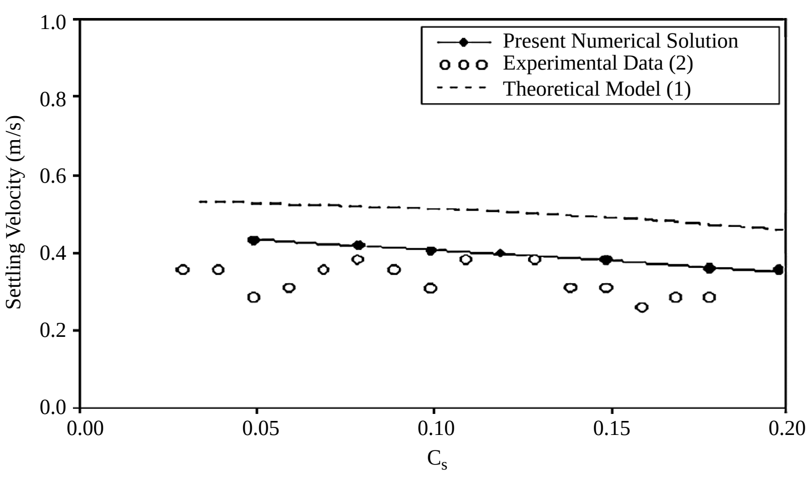


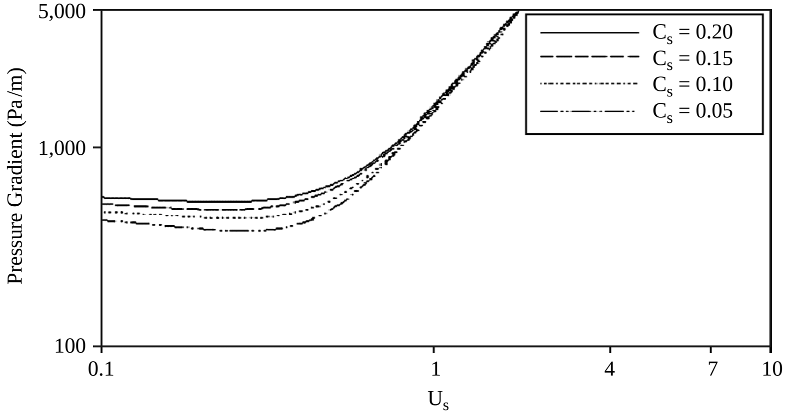

**Figure 3.**
Comparison of the terminal velocity predicted by the numerical, theoretical and the experimental results for the particles of density of $\rho = 1,240(\mathrm{kg/m^3})$ and diameter of $d_p = 3\,\mathrm{mm}$, in pipes of diameter $D = 50\,\mathrm{mm}$

high densities to prevent the creation of the stable bed and is compatible with the analytical results attained by Doron *et al.* (1986).

In Figure 7, the pressure losses caused by hydrate slurry flow for different concentrations resulted by in this paper is shown. This figure shows that pressure losses are independent of the flow velocity when there is a stable bed at the bottom of the pipe. It is also clear that an increase in the particles concentration results in an increase in the pressure losses. In spite of different data used in this work, the results have a good adoption with the results obtained by Doron and Barnea in Figure 4.

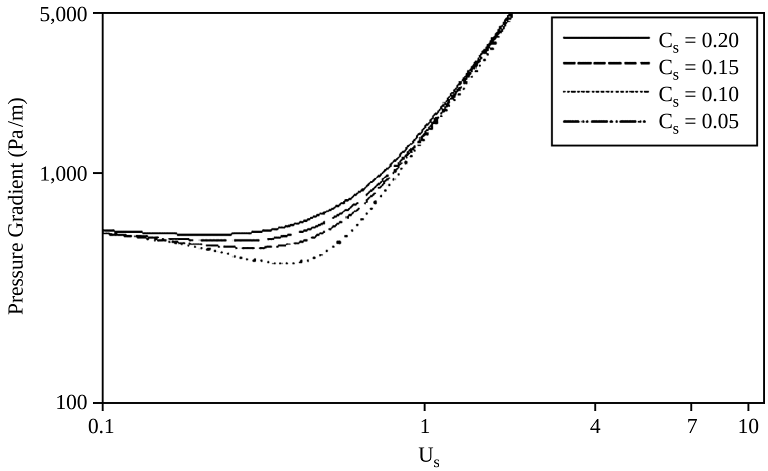
## 5. Conclusions

In recent years, flows containing solid particles have widely been used in mine industries. Different series of researches have been done on these flows and also a wide




**Source:** Using the three-layer model, Doron and Barnea (1993)

**Figure 4.** Pressure loss – flow velocity diagram for different concentrations of the slurry containing particles with the diameter of $d_p = 3$ mm and the density of $\rho_s = 1,240 (\mathrm{kg/m^3})$in pipes of $D = 50$ mm




**Source:** Using the two-layer model, Doron *et al.* (1986)

**Figure 5.** Pressure loss – flow velocity diagram for different concentrations of the slurry containing particles with the diameter of $d_p = 3$ mm and the density of $\rho_s = 1,240 (\mathrm{kg/m^3})$, in pipes of $D = 50$ mm

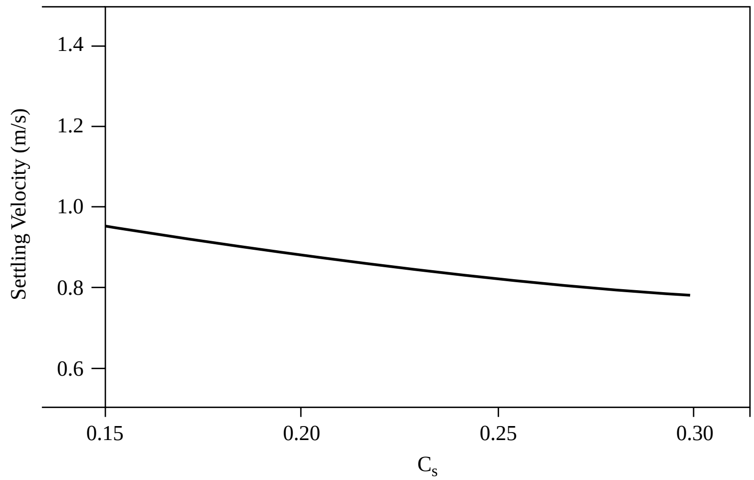



**Figure 6.**
The influence of concentration on the terminal deposit velocity for hydrate slurry containing particles with the density of $\rho^s = 2{,}420$ (kg/m$^3$) and diameter $d_p = 1.05 \times 10^{-4}$ m, in pipes of $D = 50$ mm

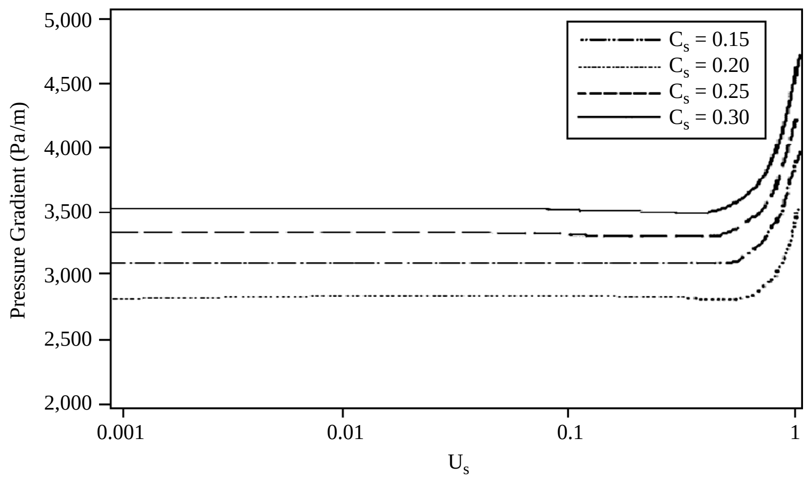



**Figure 7.**
The pressure losses of hydrate slurry for different concentrations, containing particles with diameter of $d_p = 1.05 \times 10^{-4}$ mand the density of $\rho_s = 2,420(\mathrm{kg/m^3})$, in pipes of $D = 50$ mm

range of them were done on the prediction of pressure losses in slurry flows. In this paper, using numerical methods, the related equations are solved for two- and three-layer models and the influences of the slurry velocity on the pressure losses are studied. The results show that:

- As long as there is a stable bed at the bottom of the pipe, the pressure losses are independent of terminal deposit velocity.
- Pressure losses extremely increase as the stable bed height is zero.
- Comparing slurries with water reveals that the increase in pressure losses of slurries goes through more remarkably obvious changes as terminal velocity increases.
- These changes in slurry also depend on the concentration of the slurry. As the particles concentration increases, the pressure losses considerably change.

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